# Line Loss Reduction with Distributed Energy Storage Systems 

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#### Abstract

Conventional Optimal Power Flow (OPF) minimizes line loss snapshot by controlling generation output and transformer tap position. Distributed energy storage system (DESS) that locates close to load can provide more flexible and effective control to reduce overall line loss. A dynamic optimal power flow (DOPF) method considering energy storage units is adopted to model and analyze line loss reduction by DESS. Tests are performed to validate the DOPF model and show the effectiveness of line loss reduction by DESS.


Keywords-power system, line loss, distributed energy storage system, dynamic optimal power flow

## I. Introduction

As the construction of Smart Grid is gaining momentum, the popularization of intelligent devices throughout power systems will bring about great changes in grid operations and electricity usage over the next few decades [1-3]. Changes such as controllable loads, high penetration of renewable energy sources presents power generation and consumption balancing a variety of challenges, where increasing application of energy storage units in the distribution networks will help to transform these challenges into achievements and develop system performance in the process [4-6].

Energy storage technologies contribute to the optimization of power grid operation and bring about many benefits. Bulk power energy storage systems such as pumped hydro storage and compressed air energy storage (CAES) are built up to provide off-peak base-loading for bulk power production to improve the overall performance, peaking power, frequency regulation, clean reserve generation and other kinds of ancillary service [7]. However, these large centralized storage systems are limited to appropriate sites and are generally located far away from load centers, thus can do little to optimize the energy flow in transmission and distribution networks, especially during peak demand periods.

As electric transportation and intelligent devices grow with the Smart Grid trend, the control of voltage and reliability around major load centers will face kinds of problems. On the other hand, many Smart Grid desirers expect operation performance improved by shifting the demand curve under controls or incentives. Distributed energy storage technologies will help to solve these problems in dealing with more dynamic loads and sources, which provide peak-load shaving at substations, storage of off-peak wind energy, power smoothing for solar roofs, frequency regulation, black start capability, distribution feeder reliability improvement, customer feeder load management and so on [8, 9].

This paper focuses on the line loss reduction by energy storages. Dynamic Optimal Power Flow (DOPF) considering energy storage units is used to analyze the effectiveness in line loss reduction. Power grid with distributed energy storage systems and time-varying load demand is considered in this paper, and numerical results show that line loss sees a significant reduction by energy storage devices' impact on the energy flow according to DOPF.

## II. Model and Problem Formulation

Optimal power flow (OPF) is to optimize a certain objective over controllable power system variables under certain constraints which account for the full AC network security and since Carpentier's pioneering work [10] in the 1970s, the OPF problem has drawn lots of attention and the studies on OPF has last for decades without fading [11-15]. However, OPF only considers power systems at one time snapshot and it doesn't model time-related constraints such as generation ramping constraints, fuel storage and reservoir capacity. On the other hand, the addition of storage introduces not only time-related constraints such as dynamic energy storage balances, but also an opportunity to optimize certain objective such as line loss over time horizon. In order to take energy storage into account, OPF is extended to be a Dynamic OPF (DOPF) problem which is adopted in this paper. Our goal is to optimize the storage' charge-discharge schedule to reduce the total line loss under time-varying load profile. The details are as follows.

## A. Objective Function

As a demonstration of how distributed energy storage deployment will affect line loss, the DOPF in this paper extends the objective function of OPF to minimize line loss over the whole time horizon:

$$
\begin{equation*}
\min \quad F=\sum_{t \in T} \sum_{i \in B}\left(P_{g i, t}-P_{d i, t}\right) \tag{1}
\end{equation*}
$$

where $T$ is the set of time periods of the research horizon; $B$ is the set of buses of the power system considered; $P_{g i, t}$ presents the active generation of bus $i$ at time period $t$, while $P_{d i, t}$ presents the active load.

## B. Time-separated Constraints

The following constraints should be satisfied within every single time period $t, t \in T$. Power output limits for generator $i$, $i \in G$ :

$$
\begin{align*}
& P_{g i \min } \leq P_{g i, t} \leq P_{g i \max }  \tag{2}\\
& Q_{g i \min } \leq Q_{g i, t} \leq Q_{g i \max } \tag{3}
\end{align*}
$$

where $G$ is the set of buses connected with controllable generators; $P_{g i, t}$ and $Q_{g i, t}$ are the active and reactive output of generator $i$ during time period $t$; subscript min or max denotes the minimum or maximum generation output that is available.
Power output limits of energy storage $i, i \in S$ :

$$
\begin{equation*}
P_{s i \min } \leq P_{s i, t} \leq P_{s i \max } \tag{4}
\end{equation*}
$$

where $S$ is the set of buses deployed with energy storage units; $P_{s i, t}$ is the active power output of energy storage unit $i$ at time $t$, noting that $P_{s i, t}$ can either be positive (energy storage unit is discharging ) or negative (energy storage unit is charging ). For simplicity, the MVAR contribution of these distributed energy storage units is not considered in this paper.

Capacity limits of energy storage $i, i \in S$ :

$$
\begin{equation*}
E_{s i \min } \leq E_{s i, t} \leq E_{s i \max } \tag{5}
\end{equation*}
$$

where $E_{s i, t}$ denotes the energy level, or state of charge (SOC) of energy storage unit $i$ during time period $t ; E_{s i \max }$ is the maximum capacity of energy storage and $E_{s i} \min$ is the minimum capacity, which is usually 0 . Note that $P_{s i, t}$ and $E_{s i, t}$ interact with each other, and the relations will be presented in the following.

Busbar voltage limits for node $i, i \in B$ :

$$
\begin{equation*}
V_{i \min } \leq V_{i, t} \leq V_{i \max } \tag{6}
\end{equation*}
$$

where $V_{i, t}$ is the busbar voltage of bus $i$ at time $t$.
Power-flow equations for node $i, i \in B$ :

$$
\begin{align*}
& P_{g i, t}+P_{s i, t}-P_{d i t}-V_{i, t} \sum_{j \in B} V_{j, t}\left(G_{i j} \cos \theta_{i j, t}+B_{i j} \sin \theta_{i j, t}\right)=0  \tag{7}\\
& Q_{g i, t}+Q_{s i, t}-Q_{d i, t}-V_{i, t} \sum_{j \in B} V_{j, t}\left(G_{i j} \sin \theta_{i j, t}-B_{i j} \cos \theta_{i j, t}\right)=0 \tag{8}
\end{align*}
$$

where $G_{i j}$ and $B_{i j}$ are the real and image part of the $i j$-th term of the system's busbar admittance matrix, respectively; $\theta_{i j}$ is the busbar angle difference between bus $i$ and bus $j$.

Other time-separated constraints such as system minimum spinning reserve requirement constraint, branch flow limits, taps of transformers and so on, can be included as well.

## C. Time-related constraints

Time-related constraints such as ramping rates, fuel storage and reservoir capacity are constraints for dynamic operations. Because of these dynamic operational constraints, the grid operation at certain time period may impact the later periods. As this paper focuses on distributed storage energy units, only energy storage's dynamic operational constraints are considered as follows.

Energy dynamic balance for energy storage $i, i \in S$ :

$$
\begin{equation*}
E_{s i, t}-\eta_{s i, t} P_{s i, t} T_{0}=E_{s i, t+1} \tag{9}
\end{equation*}
$$

where $t \in T$ and $T_{0}$ is the length of every single time period; $\eta_{s i, t}$ takes the value of 1 when $P_{s i, t}$ is positive (discharging) and $\eta_{s i}$ when $P_{s i, t}$ is negative (charging), where $\eta_{s i}$ represents the energy efficiency of storage $i$. Besides, the initial and end condition for energy storage $i$ is given, i.e. the SOC for the beginning of this time horizon ( $E_{s i, 0}$ ) and the beginning of next time horizon $\left(E_{s i, T+1}\right)$ is appointed according to gird operation plan.

Equations (1)-(9) form the DOPF with distributed energy storage model in this paper. An interior-point optimization software IPOPT [15] is adopted as a tool to solve the problem.

## III. Test Case

In this section we solve a test case of 3 buses and 2 time periods. The simplification in the test case provides us with a better understanding of how energy storage units affect the energy flow how the optimal charge-discharge schedule reduces the line loss. We expect to get the basic insight from this simple case and then extend to the general cases later.

The network of the test case is shown in Figure 1 and the set of buses $B$ is $\left\{b u s_{1}, b u s_{2}, b u s_{3}\right\}$. There are two load buses (bus and $b u s_{2}$ ) connected with load centers and $b u s_{3}$ is a SW bus which represents the large system, i.e. the rest of the grid. Distributed energy storage may be connected to $b u s_{1}$ or $b u s_{2}$, and the corresponding networks are shown in Figure 2, Figure 3. The voltage levels for $b u s_{1}, b u s_{2}$ and $b u s_{3}$ are $110 \mathrm{kV}, 110 \mathrm{kV}$ and 220 kV , respectively. The detailed information about the branches $\left(l_{1}, l_{2}, l_{3}\right)$ is listed in Table I. The two time periods simulate peak and off-peak loading demand of a day, and the set of time periods $T$ is $\left\{t_{\text {peak }}, t_{\text {valley }}\right\}$. Each time period $\left(T_{0}\right)$ lasts for 12 hours. The details about time periods are presented in Table II. The voltage and angle of $b u s_{3}$ are 1.0 p.u. and 0 rad for both time periods.


Figure 1. The simple system of 3 buses without energy storage.

TABLE I. Detailed Information About Branches

| Branch | From | To | Resistance <br> (p.u.) | Reactance <br> (p.u.) |
| :---: | :---: | :---: | :---: | :---: |
| $l_{1}$ | $b u s_{3}$ | $b u s_{1}$ | 0.01590 | 0.14050 |
| $l_{2}$ | $b u s_{3}$ | $b u s_{2}$ | 0.01150 | 0.11060 |
| $l_{3}$ | $b u s_{1}$ | $b u s_{2}$ | 0.00220 | 0.29150 |

TABLE II. Detailed Information About Two Time Periods

| Period | Bus1 |  | Bus2 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{P}_{\text {di,t }}$ <br> $(\boldsymbol{M W})$ | $\boldsymbol{Q}_{\text {di,t }}$ <br> $($ MVar $)$ | $\boldsymbol{P}_{\text {di,t }}$ <br> $($ MW) | $\boldsymbol{Q}_{\text {di,t }}$ <br> $($ MVar $)$ |
|  | 50 | 11 | 80 | 20 |
| $t_{\text {peak }}$ | 80 | 15 | 120 | 30 |

In order to analyze the benefits to line loss brought about by energy storage and provide a good comparison, the DOPF problem is solved for 3 situations : without energy storage, with single energy storage and with full energy storages, respectively.

## A. Without Energy Storage

In this situation there is no energy storage and the network is shown in Figure 1. Equations (1)-(3) and (6)-(8) form the DOPF problem here. The optimization solution is as follows, which is in fact the Power Flow Equation solution, as the system consists of two PQ buses and one SW bus without any other controllable methods.

The total line loss without energy storage is 52.740 MWh (15.288 MWh in period $t_{\text {valley }}$ and 37.452 MWh in period $t_{\text {peak }}$ ). As the total load demand is 3960 MWh , the line loss percentage is $1.33 \%$. The detailed solution is presented in Table III and information about branch power flows is listed in Table IV, where $P_{\text {loss }}$ represents the line loss. In the branch power flows for branch $l$ (from bus $i$ to bus $j$ ), $P_{t o}$ represents the active power flow calculated at bus $i$, while $P_{\text {from }}$ is calculated at bus $j$. The positive direction for $P_{t o}$ and $P_{\text {from }}$ is from bus $i$ to bus $j$, and the algebra sum of $P_{t o}$ and $P_{\text {from }}$ is just the line loss for branch $l . Q_{\text {to }}$ and $Q_{\text {from }}$ represent reactive power flow in the same way.

TABLE III. DOPF Solution Without Energy Storage

| Period | Bus1 |  | Bus2 |  | Bus3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{V}$ <br> $($ p.u. $)$ | $\boldsymbol{\theta}$ <br> (rad) $)$ | $\boldsymbol{V}$ <br> $($ p.u. $)$ | $\boldsymbol{\theta}$ <br> $($ rad $)$ | $\boldsymbol{P}_{\boldsymbol{g}}$ <br> $(\boldsymbol{M W})$ | $\boldsymbol{Q}_{\boldsymbol{g}}$ <br> $(\boldsymbol{\text { MVar } )}$ |
| $t_{\text {valley }}$ | 0.970 | -0.075 | 0.966 | -0.086 | 130.82 | 38.84 |
| $t_{\text {peak }}$ | 0.953 | -0.121 | 0.945 | -0.133 | 203.12 | 74.08 |

TABLE IV. Branch Power Flows Without Energy Storage

| Period | Branch | $\boldsymbol{P}_{\text {from }}$ <br> $(\mathbf{M W})$ | $\boldsymbol{P}_{\text {to }}$ <br> $(\mathbf{M W})$ | $\boldsymbol{Q}_{\text {from }}$ <br> $(\mathbf{M V a r})$ | $\mathbf{Q}_{\text {to }}$ <br> (MVar) | $\boldsymbol{P}_{\text {loss }}$ <br> $(\mathbf{M W})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{\text {valley }}$ | $l_{1}$ | 53.86 | -53.35 | 16.95 | -12.47 | 0.51 |
|  | $l_{2}$ | 77.42 | -76.65 | 25.95 | -18.57 | 0.77 |
|  | $l_{3}$ | 3.35 | -3.35 | 1.47 | -1.43 | 0.00 |
|  | $l_{1}$ | 85.07 | -83.78 | 28.93 | -17.59 | 1.29 |
|  | $l_{2}$ | 118.06 | -116.22 | 45.15 | -27.48 | 1.84 |
|  | $l_{3}$ | 3.78 | -3.78 | 2.59 | -2.52 | 0.00 |

From the above details we learn that most of the line loss takes place in $l_{1}$ and $l_{2}$ along which energy is transferred from the large system (with generations) to major load centers.

## B. With Energy Storage

Energy storage deployment is considered here which provides a method to control the energy flow to minimize the total line loss. The set of energy storage is $S=\left\{s_{1}, s_{2}\right\}$, and $s_{1}$ is connected with $b u s_{1}, s_{2}$ is connected with bus $s_{2}$. The information about energy storage is presented in Table V.

TABLE V. Energy Storage Units Infomation

| Storage | $\eta_{s i}$ | $\boldsymbol{E}_{\text {si max }} / \boldsymbol{E}_{\text {si min }}$ <br> (MWh) | $\boldsymbol{P}_{\text {si max }} / \boldsymbol{P}_{\text {si min }}$ <br> (MW) | $\boldsymbol{E}_{\text {si, }, 0} / \boldsymbol{E}_{\text {si, T+1 }}$ <br> (MWh) |
| :---: | :---: | :---: | :---: | :---: |
| $s_{1}$ | $90 \%$ | $40 / 0$ | $10 /-10$ | $20 / 20$ |
| $s_{2}$ | $92 \%$ | $60 / 0$ | $15 /-15$ | $25 / 25$ |

Firstly, we consider the situation that only one $b u s_{1}$ is connected with distributed energy storage $s_{1}$. The corresponding network is shown in Figure 2.


Figure 2. The simple system with single energy storage.
The DOPF formed in Section II is solved and the total line loss is 52.561 MWh (15.681 MWh in period $t_{\text {valley }}$ and 36.880 MWh in period $t_{\text {peak }}$ ) and the line loss percentage is $1.327 \%$. Compared with the results without energy storage, the line loss in period $t_{\text {valley }}$ increases with 0.393 MWh while the line loss in period $t_{\text {peak }}$ decreases with 0.572 MWh , resulting in that the total line loss decreases with 0.179 MWh , and decreases by $0.34 \%$. The detailed results are as follows.

TABLE VI. DOPF Solution With Single Energy Storage

| Period | Bus1 |  |  | Bus2 |  | Bus3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\boldsymbol{V}$ <br> $($ p.u. $)$ | $\boldsymbol{\theta}$ <br> $($ rad $)$ | $\boldsymbol{P}_{\boldsymbol{s}}$ <br> $(\boldsymbol{M W})$ | $\boldsymbol{V}$ <br> $($ p.u. $)$ | $\boldsymbol{\theta}$ <br> $($ rad $)$ | $\boldsymbol{P}_{\boldsymbol{g}}$ <br> $(\boldsymbol{M W})$ | $\boldsymbol{Q}_{\boldsymbol{g}}$ <br> $(\boldsymbol{M V a r})$ |
|  | 0.970 | -0.077 | -1.76 | 0.966 | -0.086 | 133.07 | 43.18 |
| $t_{\text {peak }}$ | 0.953 | -0.119 | 1.59 | 0.945 | -0.133 | 201.49 | 73.65 |

TABLE VII. Branch Power Flows With Single Energy Storage

| Period | Branch | $\boldsymbol{P}_{\text {from }}$ <br> $(\mathbf{M W})$ | $\boldsymbol{P}_{\text {to }}$ <br> (MW) | $\boldsymbol{Q}_{\text {from }}$ <br> $(\mathbf{M V a r})$ | $\mathbf{Q}_{\text {to }}$ <br> (MVar) | $\boldsymbol{P}_{\text {loss }}$ <br> (MW) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{\text {valley }}$ | $l_{1}$ | 55.19 | -54.66 | 17.11 | -12.42 | 0.53 |
|  | $l_{2}$ | 77.88 | -77.10 | 26.07 | -18.61 | 0.78 |
|  | $l_{3}$ | 2.90 | -2.90 | 1.42 | -1.39 | 0.00 |
|  | $l_{1}$ | 83.85 | -82.60 | 28.68 | -17.65 | 1.25 |
|  | $l_{2}$ | 117.64 | -115.81 | 44.98 | -27.43 | 1.82 |
|  | $l_{3}$ | 4.19 | -4.19 | 2.65 | -2.57 | 0.00 |

Next, we consider the situation that both buses are connected with energy storages. The corresponding network is shown in Figure 3 and information about energy storage units refer to Table V above.


Figure 3. The simple system with full energy storages.
The total line loss is 52.256 MWh ( 16.382 MWH at period $t_{\text {valley }}$ and 35.874 MWh at period $t_{\text {peak }}$ ). Compared with the two situations above, the line loss in period $t_{\text {valley }}$ is even higher while the value in period $t_{\text {peak }}$ is even lower, and the total line loss is reduced more significantly. The total line loss with full energy storages is $99.08 \%$ of the initial line loss without energy storage. The detailed DOPF results are listed in Table VIII and Table IX.

The details of the above two situations with energy storage show that the energy storage will charge during off-peak period and discharge during peak period, thus impact the energy flow over the time horizon, resulting in a reduction in the total line loss.

TABLE VIII. DOPF Solution With Full Energy Storages

| Period | Bus | $\boldsymbol{V}$ <br> (p.u.) | $\boldsymbol{\theta}$ <br> (rad) | $\boldsymbol{P}_{\boldsymbol{s}}$ <br> (MW) | $\boldsymbol{P}_{\boldsymbol{g}}$ <br> (MW) | $\boldsymbol{Q}_{\boldsymbol{g}}$ <br> (MVar) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{\text {valley }}$ | bus $_{1}$ | 0.970 | -0.078 | -1.69 | 0 | 0 |
|  | bus $_{2}$ | 0.965 | -0.089 | -3.05 | 0 | 0 |
|  | bus $_{3}$ | 1 | 0 | 0 | 136.11 | 43.75 |
|  | bus $_{1}$ | 0.953 | -0.118 | 1.52 | 0 | 0 |
|  | bus $_{2}$ | 0.946 | -0.130 | 2.81 | 0 | 0 |
|  | bus $_{3}$ | 1 | 0 | 0 | 198.66 | 72.85 |

table IX. Branch Power Flows With Full Energy Storages

| Period | Branch | $\boldsymbol{P}_{\text {from }}$ <br> $(\mathbf{M W})$ | $\boldsymbol{P}_{\text {to }}$ <br> $(\mathbf{M W})$ | $\boldsymbol{Q}_{\text {from }}$ <br> $(\mathbf{M V a r})$ | $\mathbf{Q}_{\text {to }}$ <br> (MVar) | $\boldsymbol{P}_{\text {loss }}$ <br> $(\mathbf{M W})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $t_{\text {valley }}$ | $l_{1}$ | 55.77 | -55.23 | 17.28 | -12.49 | 0.54 |
|  | $l_{2}$ | 80.34 | -79.52 | 26.47 | -18.56 | 0.82 |
|  | $l_{3}$ | 3.54 | -3.54 | 1.49 | -1.44 | 0.00 |
|  | $l_{1}$ | 83.31 | -82.08 | 28.45 | -17.56 | 1.23 |
|  | $l_{2}$ | 115.35 | -113.59 | 44.40 | -27.50 | 1.76 |
|  | $l_{3}$ | 3.60 | -3.60 | 2.56 | -2.50 | 0.00 |

## IV. Conclusion

This paper firstly built a DOPF model for line loss reduction by DESS. Compared to conventional OPF that reduces line loss for a single snapshot, the model in this paper can reduce the overall line loss for a whole day and therefore for longer time period e.g. a year. The newly introduced controllable variable, input/output power of DESS, brings more flexibility to optimization.

Secondly, numerical results on a test system with 3 buses and 2 time periods showed that energy storage units charge during off-peak period and discharge during peak period according to DOPF solution, thus impact the energy flow over the time horizon, resulting in a significant reduction in the total line loss.

This paper provides fundamental insight of line loss reduction by distributed energy storage systems and lays a foundation for further study on plan and operation energy storage systems.

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